Nested Modulus Sequences: Structure, Growth, and Applications

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Abstract

We introduce a class of parameterized sequences termed *Nested Modulus Sequences*, defined by a linear expression involving additive and reciprocal components. These sequences exhibit structured growth, integer subsequences, and cross-nest interactions. We analyze their asymptotic behavior, visualize their progression, and discuss applications in experimental mathematics and cryptography.

1 Definition

Let $N \in \mathbb{N}$ be the *nest index*, and $M \in \mathbb{N}$ the *modulus position*. Define the sequence $S_N = \{s_N(M)\}_{M=1}^{\infty}$ by:

$$s_N(M) = M \cdot N + \frac{M}{N} + (M - N) + (M + N) \tag{1}$$

Simplifying:

$$s_N(M) = M\left(N + \frac{1}{N} + 2\right) = M \cdot f(N) \tag{2}$$

where:

$$f(N) := N + \frac{1}{N} + 2 \tag{3}$$

2 Growth Behavior

2.1 Linear Growth in M

For fixed N, the sequence grows linearly in M with slope $f(N)$. This makes S_N a scaled arithmetic progression.

2.2 Nonlinear Dependence on N

The growth factor $f(N)$ is nonlinear in N, due to the reciprocal term $\frac{1}{N}$. As $N \to \infty$, we observe:

$$f(N) = N + \frac{1}{N} + 2 \sim N + 2 \quad (\text{since } \frac{1}{N} \to 0) \tag{4}$$

Thus:

$$s_N(M) \sim M(N+2) \tag{5}$$

2.3 Asymptotic Behavior

Fix M, then:

$$s_N(M) = M \cdot f(N) = M\left(N + \frac{1}{N} + 2\right) \tag{6}$$

As $N \to \infty$, we have:

$$s_N(M) \sim M(N+2) \tag{7}$$

As $N \to 0^+$, the reciprocal term dominates:

$$f(N) \sim \frac{1}{N} \quad \Rightarrow \quad s_N(M) \sim \frac{M}{N} \tag{8}$$

This shows a singularity at $N = 0$, and rapid growth for small N.

3 Integer Subsequence

Let $M = kN$, $k \in \mathbb{N}$. Then:

$$s_N(kN) = kN \cdot f(N) \tag{9}$$

This yields an integer-only subsequence:

$$I_N = \{i_N(k)\}_{k=1}^{\infty}, \quad i_N(k) := kN \cdot f(N) \tag{10}$$

Example: $N = 18$

$$f(18) = 18 + \frac{1}{18} + 2 = \frac{361}{18}, \quad i_{18}(k) = 361k$$

Nested Modulus Sequence for $N = 5$

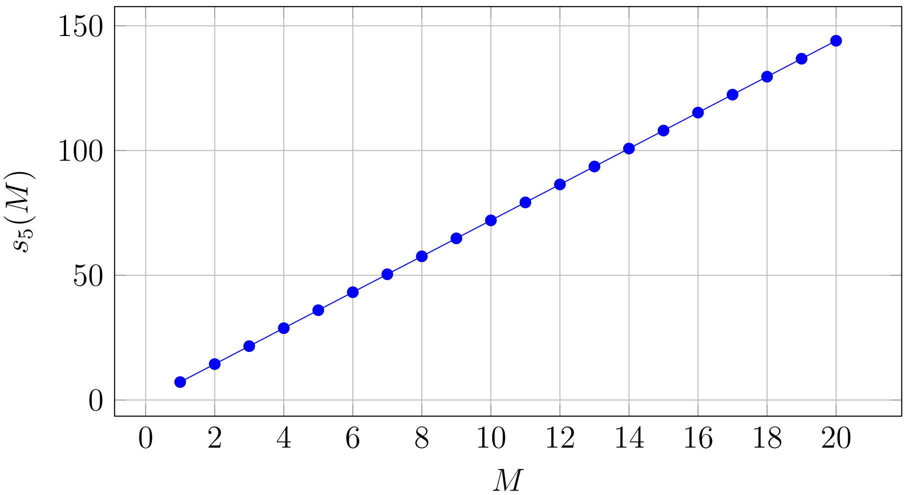

Figure 1: Linear growth of $s_5(M) = M \cdot (7.2)$

Growth Factor $f(N)$ vs. Nest Index N

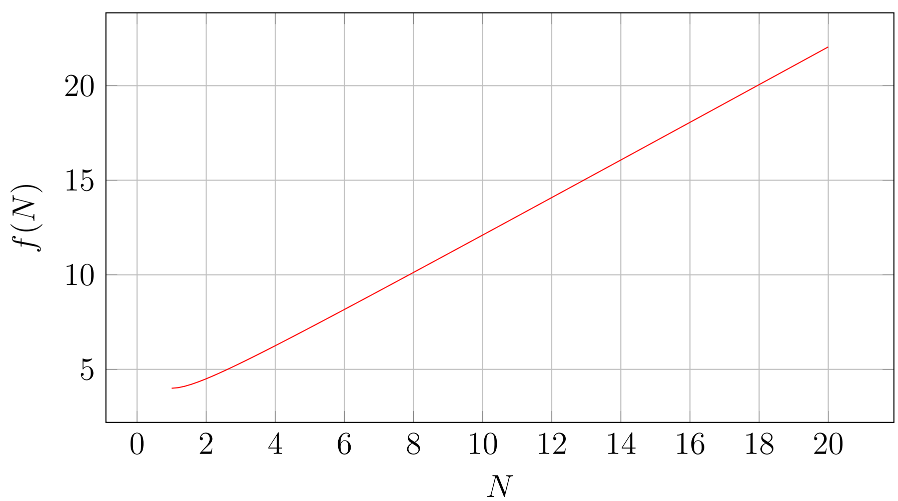

Figure 2: Nonlinear growth of $f(N)$ with asymptotic behavior

4 Visualization

5 Cross-Nest Integer Propagation

If $s_N(M) \in \mathbb{Z}$, then:

$$s_N(M) \cdot N \in \mathbb{Z} \tag{11}$$

This implies that integer values in one nest may appear in another, forming a cascade across nests. For example, if:

$$s_{18}(36) = 722 \in \mathbb{Z}, \quad \text{then 722 may appear in } S_{17}(M') \text{ for some } M'$$

6 Applications

- **Sequence Classification:** Each S_N defines a rational linear progression.
- **Cryptography:** Use $s_N(M) \mod k$ for encoding schemes.
- **Mathematical Exploration:** Study integer subsequences, growth rates, and cross-nest mappings.

7 Conclusion

Nested Modulus Sequences blend linear and nonlinear components, offering rich structure for mathematical investigation. Their predictable growth, integer subsequences, and cross-nest interactions make them promising for both theoretical and applied contexts.